

# 8.1

## Solving Equations by Graphing

### You will need

- grid paper
- a ruler
- coloured pencils

### ▶ GOAL

Use tables and graphs to solve equations.

### Learn about the Math

Carina wants to determine the area of a garden. The garden is enclosed by 32 border tiles. The garden and border tiles are part of this pattern of squares.

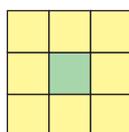


Figure 1

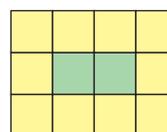


Figure 2

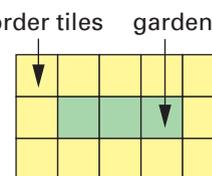


Figure 3

Carina used the variable  $n$  to represent the area of the garden. Since there are always six tiles on the left and right sides of the garden and  $2n$  tiles directly above and below the garden, the number of border tiles is  $2n + 6$ .

Carina realized that  $2n + 6$  and 32 both describe the number of border tiles. So, she wrote the **equation**  $2n + 6 = 32$ . The **solution to the equation** will tell her the area of the garden.

### equation

a mathematical statement in which the value on the left side of the equal sign is the same as the value on the right side of the equal sign; for example, the equation  $5n + 4 = 39$  means that 4 more than the product of 5 and a number equals 39

### ? How can you determine the area of a garden surrounded by 32 border tiles?

- Copy and complete the table of values using Carina's pattern rule.
- Use the data in step A to draw a scatter plot. Put the area of the garden on the horizontal axis and the number of border tiles on the vertical axis. Connect the points to form a line, and extend it.
- Use your graph to determine the area of the garden enclosed by 32 border tiles.

Area of garden (term number)	Number of border tiles (term value)
1	8
2	10
3	12
4	
5	
$n$	

### solution to an equation

the value of a variable that makes the equation true; for example, in the equation  $5n + 4 = 39$ , the value of  $n$  is 7 because  $5(7) + 4 = 39$

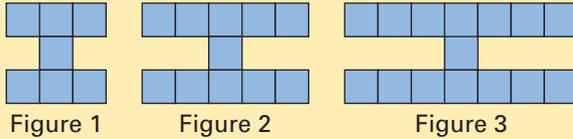
### Reflecting

- Explain why it was reasonable to extend your graphed line in step B.
- How did you use a graph to solve the equation  $2n + 6 = 32$ ?
  - How can you check your solution to make sure that it solves Carina's problem?

## Work with the Math

### Example: Using a graph to solve an equation

- a) Determine an algebraic expression for this pattern:



- b) Create and solve an equation to determine the number of the figure with 39 tiles.

#### Rishi's Solution

- a) Let  $n$  represent the figure number.

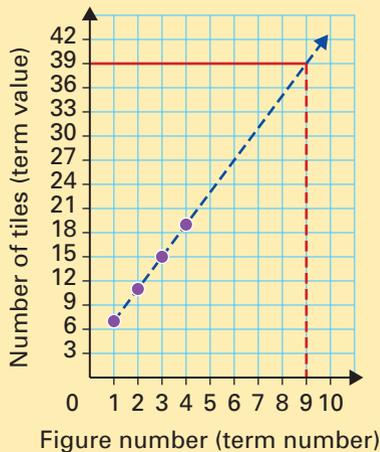
Figure number (term number)	Number of tiles (term value)
1	7
2	11
3	15
4	19
$n$	$4n + 3$

+4  
+4  
+4

I made a table of values to organize the data. I noticed that each figure has four more tiles than the previous figure. Since repeated addition is like multiplication, I know that the pattern rule includes  $4n$ .

I substituted the first few term numbers into the expression  $4n$ . The pattern rule must be  $4n + 3$ .

- b)  $4n + 3 = 39$       An H-Pattern



I needed to know the term number for a term value of 39. So, I had to solve the equation  $4n + 3 = 39$ .

I drew a graph using the data in the table. The plotted points formed a straight line, which I extended to show that the pattern continues.

I found 39 on the vertical axis. I drew a horizontal line until it touched my graphed line. Then I drew a vertical line from the intersection point to the horizontal axis. This meant that the solution to the equation is  $n = 9$ .

Figure 9 has 39 tiles.

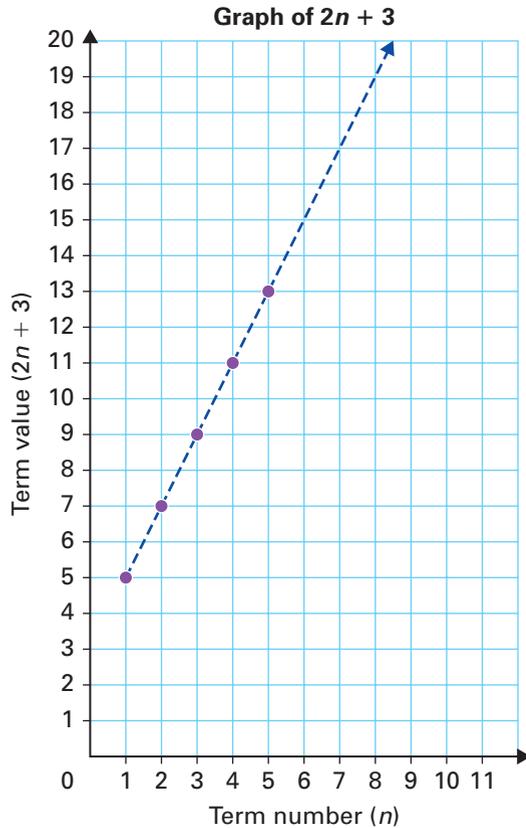
Check: Left side	Right side
$4n + 3$	39
$= 4(9) + 3$	
$= 36 + 3$	
$= 39 \checkmark$	

I checked my solution by substituting 9 for the variable  $n$ . The expressions on both sides of the equation were equal, so I knew that my solution was correct.

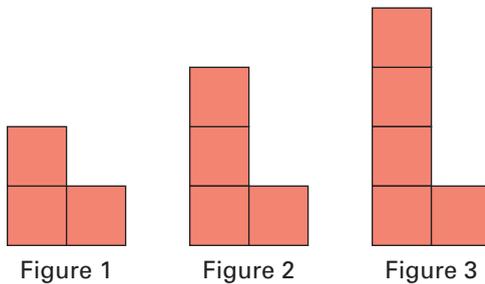


## A Checking

3. Use the graph to determine the solution to the equation  $2n + 3 = 17$ .



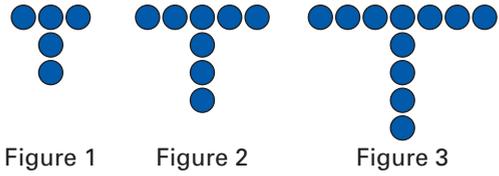
4. a) Make a table of values to represent this pattern of tiles.



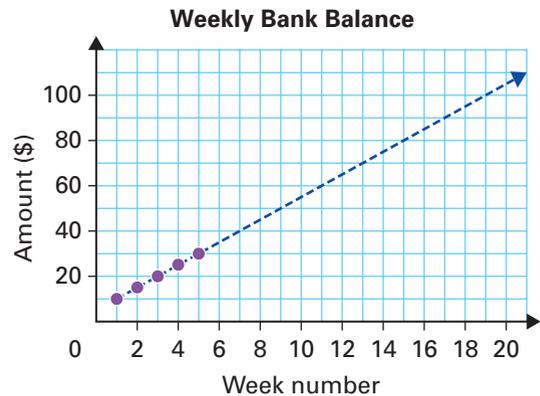
- b) Write an algebraic expression for the pattern rule.
- c) Create an equation to determine the number of the figure with 22 tiles.
- d) Draw a graph to solve your equation in part (c).

## B Practising

5. a) Make a table of values for this pattern.



- b) Write an algebraic expression for the pattern rule.
- c) Create an equation to determine the number of the figure with 23 counters.
- d) Draw a graph to solve your equation.
6. The graph shows the weekly balance in David's bank account, rounded to the nearest \$5.



- a) Make a table of values for the data in the graph.
- b) Create an algebraic expression that represents David's bank balance after  $w$  weeks.
- c) Write an equation to determine when his bank balance was \$60.
- d) Use the graph to solve your equation in part (c). After how many weeks was David's bank balance \$60?
- e) If the pattern continues, when will David's bank balance reach \$100?
- f) If the pattern continues, what will his bank balance be at the end of 20 weeks?

7. a) Examine the following pattern. What is the number of the figure that you could make using 97 toothpicks?



Figure 1      Figure 2      Figure 3

- b) Examine the following pattern. What is the number of the figure that you could make using 97 toothpicks?



Figure 1      Figure 2      Figure 3

8. On the same set of axes, use three different colours to draw a graph for the three pattern rules below. Then use your graph to determine the solutions when the term value is 19. How do the equations and solutions compare?

a)  $2n + 5$     b)  $2n + 7$     c)  $2n + 9$



9. On the same set of axes, use three different colours to draw a graph for the three pattern rules below. Then use your graph to determine the solutions when the term value is 15. How do the equations and solutions compare?

a)  $2n + 3$     b)  $3n + 3$     c)  $4n + 3$

## C Extending

10. Brooke used blue and yellow squares to represent a series of square ponds with border tiles.

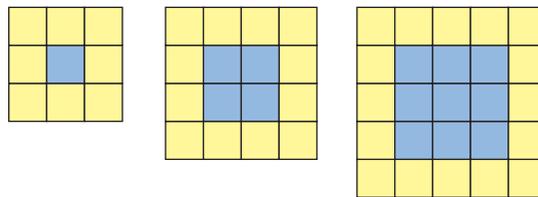


Figure 1      Figure 2      Figure 3

- a) Copy and complete the table of values for the pond pattern.

Figure number	Area of pond	Number of border tiles
1	1	8
2		
3		
4		

- b) Use the table of values to determine two algebraic pattern rules: one for the area of the pond and one for the number of border tiles.
- c) On the same set of axes, use two different colours to create a graph from the table of values.
- d) What are some differences between the pattern rules? What are some differences between the two lines on the graph?
- e) Create and solve an equation to determine the number of the figure with 56 border tiles.
- f) Create and solve an equation to determine the figure number for a pond area of 121.