2.1 Expressing Fractions as Decimals

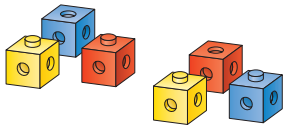
You will need
- a calculator

GOAL
Use division to express fractions as decimals.

Learn about the Math

Eva and her friends are going on a hike. Eva buys 2 kg of trail mix to share with her friends.

? How much trail mix will each hiker receive?



A. Suppose that there are three hikers. How can this picture be used to show that each hiker will get $\frac{2}{3}$ of a kilogram of trail mix?

B. Write a division statement to represent 2 kg of trail mix shared equally by three hikers.

C. Divide 2 by 3 on your calculator.

a) What is the result?

b) Why might this number be called a **repeating decimal**?

c) Complete the division 2 ÷ 3 using pencil and paper. What do you notice about the remainder after each step?

D. Suppose that there is a different number of hikers, and each will get an equal share of the 2 kg of trail mix. Copy and complete the table to show how much trail mix each hiker will get.

Number of hikers		2	3	4	5	6	7	8	9	10
Mass of trail mix each hiker gets (kg)	As a fraction		$\frac{2}{3}$							
	As a decimal									

E. Write the prime factors of each denominator.

F. Which fractions in your table have repeating decimal equivalents?

G. Which fractions have **terminating decimal** equivalents?

H. Predict which decimals will terminate and which will repeat if you extend your table to include 11, 12, 13, and so on, up to 20 hikers. Test your predictions.

repeating decimal
a decimal in which a block of one or more digits eventually repeats in a pattern; for example,
$\frac{25}{99} = 0.252\,525\ldots$
$\frac{1}{7} = 0.142\,857\,142\,857\ldots$
$\frac{31}{36} = 0.861\,111\,1\ldots$

terminating decimal
a decimal that is complete after a certain number of digits, with no repetition; for example,
$\frac{29}{40} = 0.725$

Reflecting

1. Why are "terminating" and "repeating" appropriate names for the two types of decimals?
2. How can you predict whether the decimal equivalent of a given fraction will terminate?
3. How can you predict whether the decimal equivalent of a given fraction will repeat?

Communication Tip

In repeating decimals, instead of writing the repeating digits over and over, a horizontal bar is used to mark them. For example, 0.143 514 351 435… is written as $0.\overline{1435}$.

Work with the Math

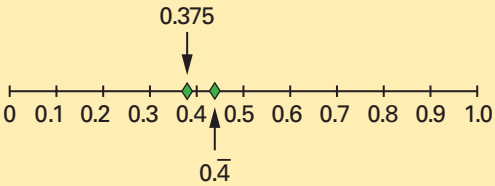
Example 1: Comparing using equivalent decimals

Susan has three bags of popcorn to share with seven other people. Nathan has four bags of popcorn to share with eight other people. If all the bags of popcorn are the same size, which group will receive larger portions of popcorn?

Eva's Solution

3 bags shared among 8 people is $3 \div 8 = 0.375$ bags.

4 bags shared among 9 people is $4 \div 9 = 0.\overline{4}$ bags.



I know that $0.\overline{4} > 0.375$. So, the people in Nathan's group will receive larger portions.

Example 2: Predicting whether a decimal will terminate or repeat

Predict whether the decimal equivalent of each fraction will terminate or repeat.

a) $\frac{7}{9}$ **b)** $\frac{53}{80}$

Ken's Solution

a) $9 = 3 \times 3$

b) $80 = 2 \times 2 \times 2 \times 2 \times 5$

I can predict whether a decimal will terminate or repeat by looking at the prime factorization of the denominator of the fraction. If the prime factors are any combination of 2s and/or 5s, the decimal will terminate. Any other prime factors mixed with the 2s and/or 5s will result in a repeating decimal.

Based on the prime factors I found, I predict that $\frac{7}{9}$ will repeat and $\frac{53}{80}$ will terminate.

$\frac{7}{9} = 0.\overline{7}$ and $\frac{53}{80} = 0.6625$

I used a calculator to check my predictions.

A Checking

4. Write each repeating decimal using bar notation.

a) 0.555 555 55…

b) 0.134 561 345 613 456…

5. Which is greater?

a) $\frac{5}{16}$ or $\frac{2}{9}$ **b)** $\frac{7}{11}$ or $\frac{5}{8}$

6. Use prime factorization to determine which fractions have decimal equivalents that repeat.

a) $\frac{3}{4}$ **b)** $\frac{5}{9}$ **c)** $\frac{9}{14}$ **d)** $\frac{19}{20}$

B Practising

7. Write each fraction as a terminating decimal.

a) $\frac{14}{25}$ **c)** $\frac{1}{16}$ **e)** $\frac{19}{20}$

b) $\frac{5}{8}$ **d)** $\frac{4}{5}$ **f)** $\frac{22}{32}$

8. Write each fraction as a repeating decimal.

a) $\frac{1}{6}$ **c)** $\frac{7}{11}$ **e)** $\frac{48}{99}$

b) $\frac{8}{9}$ **d)** $\frac{7}{15}$ **f)** $\frac{57}{111}$

9. a) Predict whether the decimal equivalent of each fraction will be a terminating or repeating decimal. Use a calculator to check your predictions.

i) $\frac{4}{9}$ **iii)** $\frac{5}{6}$ **v)** $\frac{5}{18}$

ii) $\frac{3}{5}$ **iv)** $\frac{15}{16}$ **vi)** $\frac{19}{32}$

b) Order the decimals from least to greatest.

10. a) Express each fraction in the first three terms of this pattern as a repeating decimal.

$\frac{1}{11}, \frac{2}{11}, \frac{3}{11} \ldots$

b) Describe the pattern.

c) Use the pattern to predict the decimal equivalents for $\frac{6}{11}$ and $\frac{9}{11}$.

11. Use >, <, or = to make each statement true.

a) $0.2 \square 0.\overline{2}$

b) $\frac{45}{99} \square 0.\overline{45}$

c) $0.\overline{82} \square \frac{4}{5}$

d) $\frac{6}{11} \square \frac{7}{13}$

e) $0.357\ 357\ 357\ldots \square 0.\overline{375}$

12. Order the numbers from least to greatest.

a) $\frac{1}{8}, \frac{5}{7}, 0.35, 0.\overline{39}, \frac{9}{10}$

b) $0.56, 0.5\overline{6}, 0.\overline{56}, \frac{5}{9}, \frac{27}{50}$

C Extending

13. Write each decimal as a fraction.

a) 0.1625 **c)** 0.272 727…

b) 0.0777… **d)** 0.065 656 5…

14. Use the fact that $\frac{1}{3} = 0.3333\ldots$ to predict the decimal equivalent for each fraction.

a) $\frac{2}{3}$ **c)** $\frac{1}{30}$

b) $\frac{1}{9}$ **d)** $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

15. Determine fractions of the form $\frac{1}{\square}$, in which the digits after the first two digits after the decimal place repeat. What do the denominators have in common?