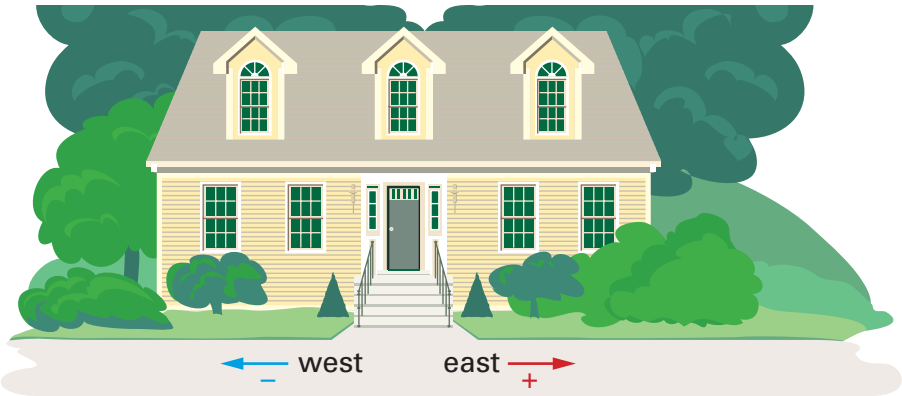
6.4 Multiplying Integers

You will need
- number lines
- coloured pencils
- red and blue counters
- a calculator

GOAL
Develop and apply strategies to multiply integers.

Learn about the Math

Eva says, "I can hike about 3 km/h. Sometimes I walk east from my home. Other times I walk west."



? If you know where Eva is now, how can you calculate where she was earlier?

Use a positive sign to mean hiking to the east. Use a negative sign to mean hiking to the west. Home is neither positive nor negative because it is located at 0.

A. How far from home is Eva if she hikes east for 10 h at 3 km/h? Draw this on a number line.

B. Describe your drawing in step A using a repeated addition sentence. Then describe your drawing using a multiplication sentence.

C. How far, and in which direction, has Eva hiked if her location can be determined using each of these calculations?
a) $4\text{ h} \times 3\text{ km/h}$ **b)** $2\text{ h} \times (-3\text{ km/h})$

D. Model each multiplication in step C on a number line.

E. Suppose that Eva has been hiking east for 6 h.
a) Why does $6\text{ h} \times 3\text{ km/h}$ represent her distance from home?
b) Why does $0 - 6\text{ h} \times 3\text{ km/h}$ represent how far, and in which direction, she would need to hike to get back home?
c) Why does $0 - 6\text{ h} \times 3\text{ km/h}$ have the same result as $-6\text{ h} \times 3\text{ km/h}$?

F. Model each multiplication in step E on a number line.

G. Suppose that Eva has been hiking west for 2 h.

a) Why does $2\text{ h} \times (-3\text{ km/h})$ represent her distance from home?

b) Why does $0 - 2\text{ h} \times (-3\text{ km/h})$ represent how far she would need to hike to get back home?

Reflecting

1. Why does it make sense to describe east as positive (+) and west as negative (−)?
2. How does knowing $0 - 18 = -18$ show that $(-6) \times 3 = -18$ in step E?
3. How does knowing $0 - (-6) = 6$ show that $(-2) \times (-3) = 6$ in step G?
4. How can you predict the sign of the product of two integers?

Communication Tip

The multiplication of integers can be written without the multiplication symbol (×). For example, $-10 \times (-20)$ can be written as $(-10)(-20)$.

Work with the Math

Example 1: Using integer products to solve distance problems

Eli walks to the west at 80 m/min.

a) Where will he be after 5 min?

b) Suppose that Eli walked west for 18 min. His position would be $18\text{ min} \times (-80\text{ m/min})$. How far must he walk to return to his starting position? What direction must he walk?

Denis's Solution

a) $5\text{ min} \times (-80\text{ m/min}) = -400$

Eli will be 400 m west.

I multiplied $5 \times 80 = 400$.

I know that a positive integer multiplied by a negative integer results in a negative integer. So the product must be −400.

b) $18\text{ min} \times (-80\text{ m/min}) = 18 \times (-80)$

$= -1440$

Eli must walk 1440 m east.

I multiplied $18 \times 80 = 1440$.

I know that a positive integer multiplied by a negative integer results in a negative integer. So the product must be −1440.

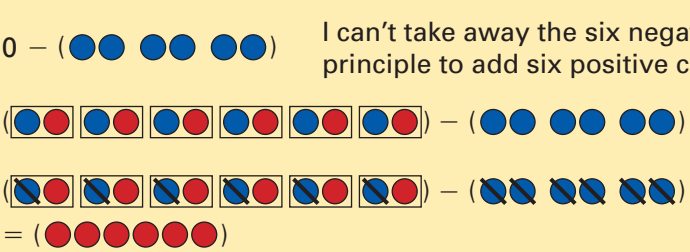
Eli walked 1440 m west. He must walk back the same distance, but in the opposite direction, to return to his starting position.

Example 2: Modelling integer multiplication

Multiply $-3 \times (-2)$ using a model.

Eva's Solution: Using counters

I can model $3 \times (-2)$ by adding three groups of two negative counters to 0.
So, I can model $-3 \times (-2)$ by subtracting three groups of two negative counters from 0.

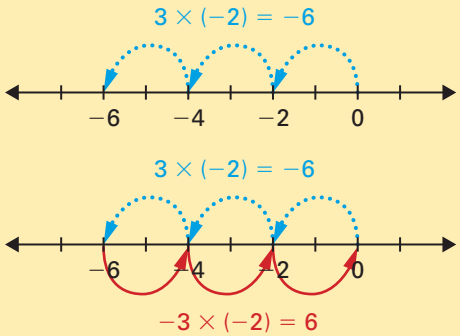


I can't take away the six negative counters. So I used the zero principle to add six positive counters and six negative counters.

Then I subtracted.

Six positive counters are left.
The answer is $-3 \times (-2) = 6$.

Nathan's Solution: Using a number line



$-3 \times (-2)$ means the same as $0 - 3 \times (-2)$, which is the opposite of $3 \times (-2)$.

To show $3 \times (-2)$, I drew 3 dotted blue arrows going left from 0, with each arrow 2 units. The arrows stop at -6.

To show the opposite of $3 \times (-2)$, I drew 3 solid red arrows going right from -6 back to 0.

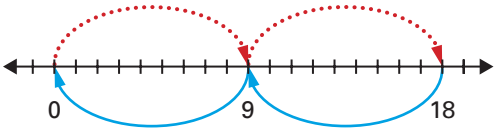
These red arrows go to the right 6 units. So the answer must be 6, which is the opposite of -6.

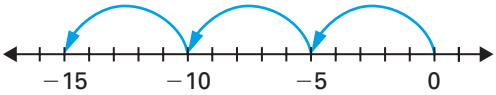
A Checking

5. Write the multiplication sentence that each model represents.

a) 

b) 

c) 

d) 

6. Dario is on a cycling trip. He started at 0 km. He is now at position $20 \text{ h} \times (-20 \text{ km/h})$. When did he reach each of the following positions? Explain your reasoning.

a) $10 \text{ h} \times (-20 \text{ km/h})$

b) $8 \text{ h} \times (-20 \text{ km/h})$

c) 0 km

B Practising

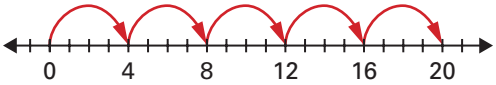
7. a) Model $-4 \times (-3)$ on a number line. Calculate the product. Explain what you did.

b) Model $-4 \times (-3)$ with counters. Calculate the product. Explain what you did.

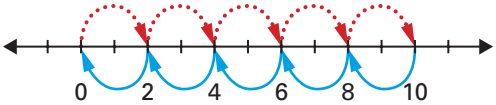
8. How would you calculate $(-4)(-7)$? Justify your strategy.

9. Write each as a multiplication expression, and then solve.

a)



b)



10. Write an integer multiplication sentence for each description.

a) Tyler rode a bus west for 4 h at 100 km/h.

b) Jenna babysat for 3 h, earning $5 an hour.

c) The temperature fell 2°C a day for 6 days.

11. Jasmine has 50 shares of a stock. The value of each share went down by $2 today. Express the total change in value of Jasmine's shares as an integer calculation.

12. Write a multiplication question for each repeated addition, and then solve.

a) $-5 + (-5) + (-5)$

b) $-8 + (-8) + (-8) + (-8) + (-8)$

c) $0 + 0 + 0 + 0 + 0 + 0 + 0$

13. Write a repeated addition question for each multiplication, and then solve.

a) $2 \times (-9)$

b) $(4)(8)$

c) $(3)(-6)$

d) $(7)(0)$

14. Multiply.

a) -2×4

b) $-8 \times (-9)$

c) $(7)(8)$

d) $(-9)(-9)$

15. Calculate each product. Order the products from greatest to least.

a) $(0)(-20)$

b) $(-6)(-30)$

c) $(7)(-80)$

d) $(-20)(50)$

16. What is the greatest possible product of any two numbers in this list? Explain your answer.

$-3, -7, -15, 6$

17. The product of two integers is between -20 and -25. What are the integers? Give five answers.

18. Determine the missing integer for $-9 \times (\square) = 63$. Explain what you did.

19. Continue each pattern for the next three terms. Explain the pattern rule.

a) $-2, 4, -8, 16, \square, \square, \square, \ldots$

b) $-15, 30, 90, -360, \square, \square, \square, \ldots$

20. Multiply.

a) $-5 \times 3 \times (-2)$

b) $-2 \times (-3) \times (-4) \times (-5)$

c) $4 \times (-3) \times (-2)$

21. The product of five different integers is -80.

a) What is the least possible sum of these five integers?

b) Is it possible for the product of four different integers to be -80? Explain.

22. Estimate each product.

a) $-35 \times (-25)$

b) $(-18)(38)$

c) $-21 \times 9 \times (-16)$

C Extending

23. Multiply any three integers that are not in the same row or column. Repeat with other sets of three integers. What do you notice?

−32	40	−24
28	−35	21
−8	10	−6