

# 1.1

## Identifying Prime and Composite Numbers

You will need

- a calculator
- grid paper

### ▶ GOAL

Determine whether a number is prime or composite.

### Learn about the Math

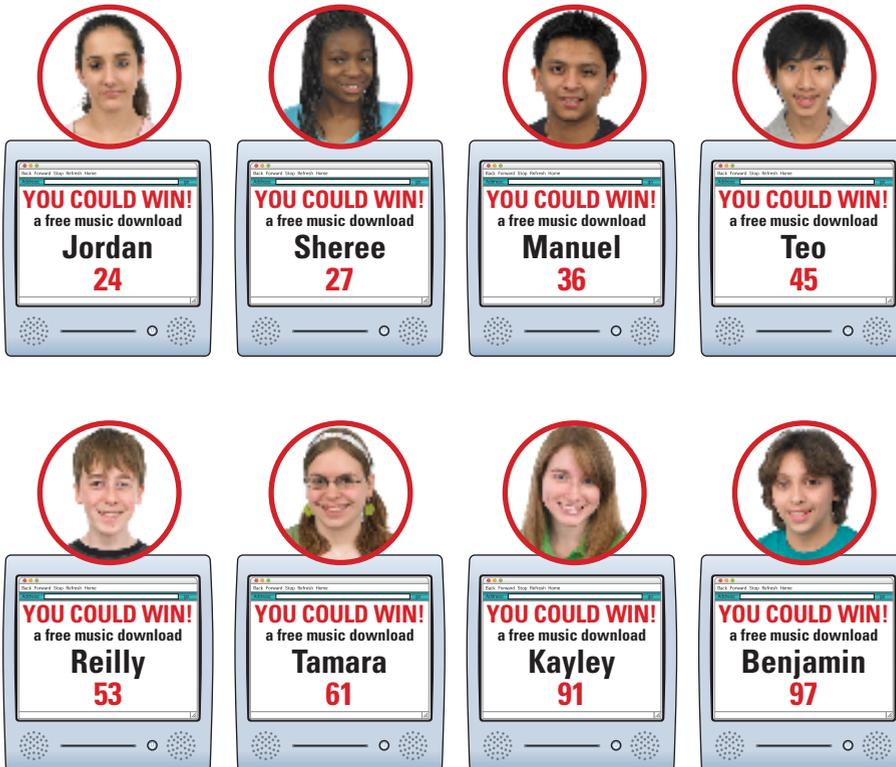
Jordan and her friends have discovered a new Web site that sells legal music downloads. Each of the first 100 customers will be randomly assigned a number from 1 to 100 for a chance to win a prize. Customers who receive a **prime number** will win a free music download. Customers who receive a **composite number** will win nothing.

#### prime number

a number that has only two factors, 1 and itself; for example, 17 is a prime number because its only factors are 1 and 17

#### composite number

a number that has more than two factors; for example, 12 is a composite number because its factors are 1, 2, 3, 4, 6, and 12



### ? Which students will win a free music download?

- Identify all the factors of 24, 27, 36, and 45.
- How do your results in step A show that none of the students who received these numbers will win a free music download?

- C. Will any student who receives an even number other than 2 win a free music download? Explain your reasoning.
- D. Will any student who receives a multiple of 3 other than 3, or a multiple of 5 other than 5, win a free music download? Explain your reasoning.
- E. Tamara received the number 61. To determine if 61 is a composite number, why would you divide 61 by prime numbers only?
- F. Which students will win a free music download?

## Reflecting

1. Which digits cannot be the last digit of a prime number greater than 10?
2. What steps would you follow to determine whether a number is prime?

## Work with the Math

### Example: Using divisibility rules to determine factors

Is 187 a prime number or a composite number?

#### Jordan's Solution

2 is not a factor of 187 because 187 is an odd number.

I used divisibility rules to see if prime numbers from 2 onward are factors of 187.

3 is not a factor of 187 because the sum of the digits is  $1 + 8 + 7 = 16$ , and 3 is not a factor of 16.

5 is not a factor of 187 because the last digit is not 0 or 5.

$$\begin{array}{r} 26.71 \\ 7 \overline{)187} \end{array}$$

7 is not a factor of 187 because the quotient is not a whole number.

$$\begin{array}{r} 17 \\ 11 \overline{)187} \end{array}$$

11 is a factor of 187 because the quotient is a whole number.  
187 is a composite number.

I stopped testing for prime factors after I divided by 11 because I knew that 187 has more than two different factors. It has 1, 11, 17, and 187 as factors.



## A Checking

3. Some students received numbers from 70 to 80 in the music download contest. Which students will win a free download? Explain your reasoning.

4. a) Examine the numbers listed below. Identify the only number that might be prime. Explain how you know that the other numbers are composite.

163  
23 452  
1 000 000  
123 123 123  
2175

b) Explain how you can prove that the number you identified is prime.

## B Practising

5. Identify each number as prime or composite. If the number is composite, list all of its factors.

- |       |        |
|-------|--------|
| a) 17 | e) 67  |
| b) 25 | f) 99  |
| c) 47 | g) 161 |
| d) 48 | h) 171 |

6. Which numbers between 30 and 40 are prime?

31 32 33 34 35 36 37 38 39

7. Show that there are no prime numbers from 200 to 210.

200 201 202 203 204 205  
206 207 208 209 210

8. How do you know that the product of any two numbers greater than 1 must be a composite number?

9. If you add a prime number to itself, is the sum composite or prime? Explain your reasoning.

10. a) Write the numbers 1 to 60 on grid paper, in rows of six. Circle the prime numbers.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18

b) What do you notice about the locations of the prime numbers greater than 3?

11. The consecutive numbers 2 and 3 are both prime. How do you know that there are no other consecutive prime numbers?

12. Twin primes are pairs of primes that differ by 2. The first twin primes are 3 and 5. List all the twin primes less than 100.



13. The area of a rectangle is  $991 \text{ cm}^2$ . If 991 is a prime number, what are the whole number dimensions of the rectangle? Explain your reasoning.

14. A classroom of students can be divided into two, three, and five groups, with no students left over. How many students are likely in the class?

15. Explain why there are no square prime numbers.

16. What prime numbers do you get when you enter each number from 0 to 10 in this expression?

$$\square^2 + \square - 1$$

17. The number 123 123 123 123 123 12  $\square$  is divisible by 3 and 9. What is the missing digit? Explain your reasoning.

## C Extending

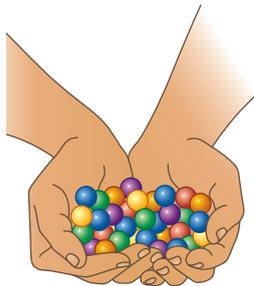
18. You get prime numbers when you enter the numbers from 1 to 40 in the expression  $\square^2 - \square + 41$ .

For example,  $8^2 - 8 + 41 = 97$ .

Explain why this expression does not give a prime number when you enter 41.

19. When the marbles in a bag are divided evenly between two friends, there is one marble left over. When the same marbles are divided evenly among three friends, there is one marble left over. When the marbles are divided evenly among five friends, there is one marble left over.

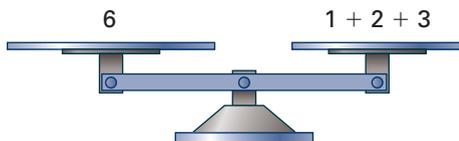
- a) What is the least possible number of marbles in the bag?
- b) What is another possible number of marbles in the bag?



20. What is the least number that is divisible by 2, 3, 4, 5, and 7? Explain your reasoning.

21. A number is **perfect** if all of its factors, other than the number itself, add up to the number. For example, the factors of 6 are 1, 2, 3, and 6. Since  $6 = 1 + 2 + 3$ , 6 is a perfect number.

- a) Why is no prime number a perfect number?
- b) Show that 496 is a perfect number.
- c) There is one perfect number greater than 6 and less than 50. Determine the number.



22. Suppose that you and a partner are playing a game with two dice. You roll the dice and add the numbers. You get 1 point if the sum is a prime number. Your partner gets 1 point if the sum is a composite number. Who is more likely to win? Explain your reasoning.

I rolled 5 and 3.  
The sum is 8. That's a composite number, so my partner gets a point.



23. In a different game, you roll two dice and multiply the numbers. You get 1 point if the product is a prime number. Your partner gets 1 point if the product is a composite number. Who is more likely to win? Explain your reasoning.

